Subgap anomaly and above-energy-gap structure in chains of diffusive SNS junctions

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We present the results of low-temperature transport measurements on chains of superconductor–normal-constriction–superconductor (SNS) junctions fabricated on the basis of superconducting PtSi film. A comparative study of the properties of the chains, consisting of 3 and 20 SNS junctions in series, and single SNS junctions reveals essential distinctions in the behavior of the current-voltage characteristics of the systems: (i) the gradual decrease of the effective suppression voltage for the excess conductivity observed at zero bias as the quantity of the SNS junctions increases, (ii) a rich fine structure on the dependences $dV/dI - V$ at dc bias voltages higher than the superconducting gap and corresponding to some multiples of $2\Delta/e$. A model to explain this above-energy-gap structure based on energy relaxation of electron via Cooper-pair-breaking in superconducting island connecting normal metal electrods is proposed.

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In the past few years, the mesoscopic systems, consisting of a normal metal (N) or heavily doped semiconductor being in contact with a superconductor (S), have attracted an increased interest mainly because of the richness of the involved quantum effects [1]. The key mechanism governing the carrier transport through the NS contact is the Andreev reflection [2]. In this process, an electron-like excitation with an energy ϵ smaller than the superconducting gap Δ moving from the normal metal to the NS interface is retro-reflected as a hole-like excitation, while a Cooper pair is transmitted into the superconductor. This phenomenon is the basis of the proximity effect, which generally implies the influence of a superconductor on the properties of a normal metal when being in electrical contact. The consequences of Andreev reflection on the current-voltage characteristics (CVC) of a NS junction are studied in detail in the so-called BTK theory [3], which describes the subgap current transport in terms of ballistic propagation of quasiclassical electrons through the normal-metal region, accompanied by Andreev and normal reflections from NS interface. The probability of Andreev reflection and normal reflection are energy-dependent quantities and the relation between them depends on the barrier strength at NS interface, that is characterized by a parameter Z ranging from 0 for a perfect metallic contact to ∞ for a low transparency tunnel barrier. For a perfect contact ($Z = 0$), probability of Andreev reflection is equal to unity for particles with an energy ϵ smaller than the superconducting gap Δ and the subgap conductance are found to be twice the normal state conductance thus demonstrating the double charge transfer. In the other limit, when $Z \to \infty$, the conductivity is very small.

When a normal metal is placed between two superconducting electrodes another mechanism is involved in charge transfer. It is multiple Andreev reflection process (MAR). The concept of MAR was first introduced by Klapwijk, Blonder and Tinkham [4] in order to explain the subharmonic energy gap structure observed as dips in the differential resistance dV/dI of SNS junctions at the voltages $V_n = 2\Delta/ne$ ($n = 1, 2, 3, \cdots$). In all systems, diffusive as well as ballistic, the MAR mechanism relies on the quasiparticles to be able to add up energy gained from multiple passages. This energy gain results in strong quasiparticle nonequilibrium. Calculations of the CVC, within the approach developed in Ref. [3], and taking into account MAR process was performed in [5] (OTBK theory). It was shown that MAR process results not only in SGS at voltages $V_n = 2\Delta/ne$ but affects the general form of CVC at all voltages as well. Although BTK theory is not intended to describe diffusion transport in normal-metal region, a suitable value $Z \sim 0.55$ of the barrier strenght gives result very close to those obtained by Green's-function method for microconstriction in the dirty limit [6]. Experimentally, in some cases BTK theory has been successful in providing a method to extract the junction reflection coefficients based on experimental CVC [7,8].

At present, a large variety of SNS junctions (most of them are diffusive) is being fabricated and studied [7–14]. The investigations of these junctions are primarily focused on the nonlinear behavior of the current-voltage characteristics, which exhibit the anomalous resistance dip at zero bias [zero bias anomaly (ZBA)], the subharmonic energy gap structure (SGS), etc. These experiments have revealed a number of peculiar properties of diffusive SNS junctions, the explanation of those being beyond the ballistic BTK and OTBK theories. Nowadays the properties of diffusive SNS junctions are considered to be determined not only by the parameters of the NS interface, but nonlocal coherent effects as well, namely (i) the superposition of multiple coherent scattering at the NS interfaces in the presence of disorder (so-called reflectionless tunneling [15]) and (ii) electron-electron interaction in the normal part. The latter is one of the important points of a recently developed "circuit theory" when applied to diffusive superconducting hybrid systems [16]. Within this approach, which is

based on the use of nonequilibrium Green's functions, the electron-electron interaction induces a weak pair potential in the normal metal. It results not only in a change of the resistance, but also in a non-trivial distribution of the electrostatic and chemical potentials in the structure, which implies non-local resistivity. Following the spirit of Nazarov's circuit theory Bezuglyi *et al.* [17] have developed a consistent theory of carrier transport in long diffusive SNS junctions with arbitrary transparency of NS interfaces. Although much work both theoretical and experimental has been done on single SNS junctions, it is a challenge to fabricate and carry out comparative measurements on multiply connected SNS systems. In this paper, we present the results of low-temperature transport measurements on chains of SNS junctions and on single SNS junctions and perform the comparative analysis of their properties.

The design of our samples is based on the fabrication technique of SNS junctions that has been recently proposed and realized by us for preparation of single SNS junctions and two-dimensional arrays of SNS junctions [13,14]. The main idea employed in those experiments was to use superconductive and normal region made of the same material. The point is the suppression of superconductivity in the submicron constrictions made in an ultrathin polycrystalline PtSi superconducting film by means of electron beam lithography and subsequent plasma etching. Here we use the same fabrication technique to prepare chains of SNS junctions.

The original PtSi film (thickness – 6 nm) was formed on a Si substrate. The film was characterized using Hall bridges 50 μm wide and 100 μm long. The film had a critical temperature T_c = 0.56 K. The resistance per square was 104 Ω. The carrier density obtained from Hall measurements was $7 \cdot 10^{22}$ cm^{-3}, corresponding to a mean-free path l = 1.2 nm and a diffusion constant D = 6 cm^2/s, estimated using the simple free-electron model.

Single SNS junction is a constriction between two holes made in the film and placed in the Hall bridge. The hole diameters are 1.7 μm and the distance between centres of holes is 2.1 μm, resulting in a width of narrowest part of the constriction of 0.4 μm. A scanning electron micrograph and a schematic view of a sample are presented in Fig.1a,b. The chains of SNS junctions are a series of constrictions connected by the islands of the film (Fig.1c,d). They are designed to provide the possibility of the comparative study. The dimensions of constrictions are identical with those of the single SNS junction, and the characteristic size of the islands of the film is ~ 1.3 μm. As the constrictions are not superconducting we have a chain of SNS junctions (Fig.1e).

The measurements were carried out with the use of a phase sensitive detection technique at a frequency of 10 Hz that allowed us to measure the differential resistance (dV/dI) as a function of the dc current (I). The

ac current was equal to 1–10 nA. Figure 2 shows typical dependences of dV/dI-V for the samples with single constriction (1D-1) and chains, consisting of 3 (sample 1D-3) and 20 (sample 1D-20) SNS junctions in series. On the abscissa the value of voltage obtained by numerical integration of the experimental dependences dV/dI-I and afterward divided by the number of SNS junctions is plotted. Measured differential resistance is also divided by the number of SNS junctions (by 3 for structure 1D-3 and by 20 for structure 1D-20). Thus for the chains data presented in Fig.2 are dependences the differential resistance as a function of dc bias voltage per SNS junction in the average. As is seen at high voltage for chains the average resistances of SNS unit of chains obtained in such a way are close to each other and to the value dV/dI of the structure with single SNS junction. A common feature for all structures studied is a minimum of the differential resistance at zero bias voltage. It points out the high transparency of NS interfaces in these samples. The data for single SNS junction (1D-1) exhibit a behavior very similar to that reported in our previous work [13]. In Ref. [17] the differential resistance of long diffusive SNS junction with perfect NS interfaces at zero voltage was estimated by the following expression:

$$dV/dI(0) = R_N(1 - 2.64\xi_N/L), \qquad (1)$$

where $\xi_N = \sqrt{\hbar D/2\pi kT}$ is the decay lenght for the pair amplitude, L is the lenght of the normal-metal region, and R_N is its resistance at $T > T_c$. The expression (1) has obtained on condition that the inelastic lenght l_ϵ exceeds the junction lenght L. The energy relaxation is described by the time $\tau_\epsilon^{-1} = \pi\epsilon^2/(8\hbar E_F)$. This is the time it takes for a "hot" quasiparticle with energy ϵ much larger than temperature T to thermalize with all the other electrons. For the excitation energy $\epsilon = 2\Delta$ ($\Delta/e \sim 270$ μV) we obtain $l_\epsilon = \sqrt{D\tau_\epsilon} \sim 4.6$ μm. The magnitude of l_ϵ really exceeds the characteristic lenght of the constriction. As R_N we take the difference between the resistance of the whole structure with constriction and the resistance of the original film without constriction measured between the same probes at $T > T_c$. It gives $R_N \sim 530$ Ω. Assuming $L \sim 1$ μm, we find from (1) at T = 100 mK the differential resistance at zero bias $dV/dI(0) \sim 0.8R_N = 420$ Ω. As is seen in Fig.2, this value is close fit to measured $dV/dI(0)$ for the sample with single constriction 1D-1. A clear-cut distinction between dependences dV/dI-V of the single SNS junction and the chains consists in non-monotone behavior of dV/dI-V characteristics of the latter. The differential resistance of the chains has a minimum at zero bias voltage and shows a maximum at a finite bias voltage of about 250 μV for the sample 1D-3 and 30 μV for 1D-20. The second feature is the gradual decrease of the effective suppression voltage for the excess conductivity observed at zero bias as the quantity of the SNS junctions increases. The same behavior – non-monotonic dV/dI-V

characteristics and the decrease of the effective suppression voltage for the excess conductivity in comparison of that in single SNS junctions – has been observed in two-dimensional arrays of SNS junctions [14].

The most unexpected result obtained for chains of SNS junction is the presence of a fine and fully symmetrical structure as dips in the differential resistance at high biases (Fig.3a,b). The positions of these dips approximately correspond to 4, 6, and 12 multiples of the superconducting gap. As is seen in Fig.3c, the temperature dependence of the positions of these dips actually reflects the dependence $\Delta(T)$. To our knowledge the results presented in this paper is the first investigation of chains of SNS junctions and the first observation such above energy gap structure (AGS). To offer an explanation of the AGS let us consider a current driven in the normal region at high voltages. As the inelastic lenght more than the normal metal region, at finite voltage quasiparticle passing through N_1 between two superconductors S_1 and S_2 (Fig.4b,d) results in a nonequilibrium energy distribution of quasiparticles. For a start we address this problem to the ballistic OTBK theory [5]. In this approach the quasiparticles are divided into two subpopulation which depend on their direction of motion $f_{\rightarrow}(E)$ and $f_{\leftarrow}(E)$, with all energies measured with respect to the local chemical potential. The current through the junction is

$$I \propto \int_{-\infty}^{+\infty} [f_{\rightarrow}(E) - f_{\leftarrow}(E)]dE. \qquad (2)$$

In Fig.4a,c we have plotted $f_{\rightarrow}(E)$, $f_{\leftarrow}(E)$, and $[f_{\rightarrow}(E) - f_{\leftarrow}(E)]$, assuming a parameter $Z = 0.55$. The voltage across the junction equals $4\Delta/e$ and $6\Delta/e$ in Fig.4a and Fig.4c, respectively. The function $[f_{\rightarrow}(E) - f_{\leftarrow}(E)]$ is depicted by the blackness (high values are black) in Fig.4b,d, with the energy scale being plotted vertically. These calculations clearly show that the nonequilibrium distributions are sharply peaked at the four gap edges. What happens to quasiparticle injected to the superconducting electrode S_2 and having the energy more than 2Δ measured with respect to the gap edge of S_2? They can decay under spontaneous phonon emission into states of lower energy [18]. Phonons emitted in the process of energy relaxation of injected quasiparticles can be reabsorbed via Cooper-pair-breaking. Depending on the volume of finite states, the probability of this process is non-monotone function of the primary energy of injected quasiparticles. It takes the peak values at the energies multipled to 2Δ, for density of quasiparticle states to have the singularity at the gap edge. As a result one quasiparticle injected to S_2 with energy high the gap edge by $m \cdot 2\Delta$ can cause up to $1+2m$ quasiparticles injected to N_2 (Fig.4b,d) and consequently the increase of the differential resistance. The realization of this scenario requires the lenght of energy relaxation to be comparable to dimensions of superconductor. It is really fulfilled in the

SNS systems studied. Proposed mechanism supposes the sistem consisting of two SNS junctions in series to be sufficient in order to observe the AGS.

In summary, we performed the first comparative study of the properties of the chains and single SNS junctions. Our experiments reveal essential distinctions in the behavior of the current-voltage characteristics of these systems. A detailed quantitative analysis needs to take into account the contributions of nonlocal coherent transport and an effect of quasiparticle injection in both the superconducting and normal regions of chains of the SNS junctions.

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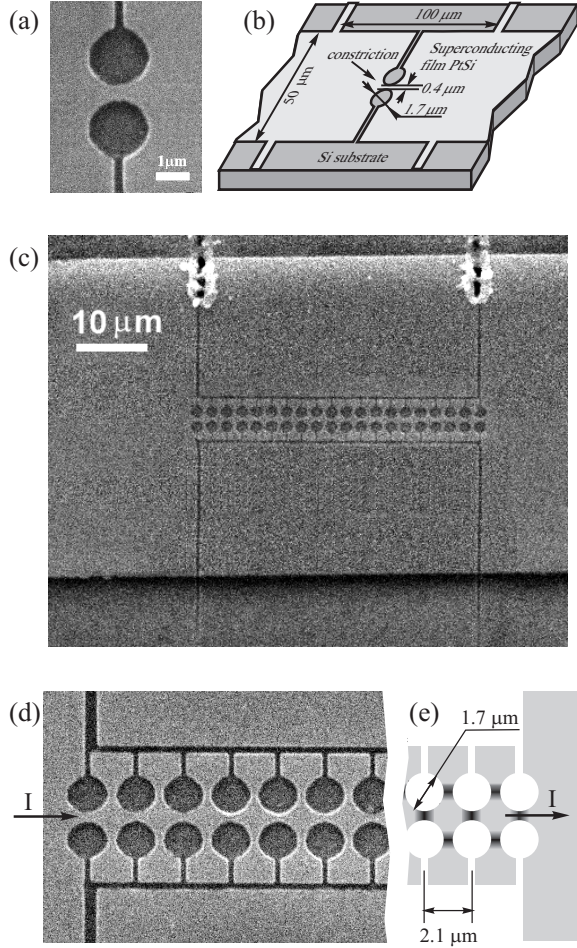
FIG. 1. (a) Scanning electron micrograph of the sample with a single constriction formed by electron beam lithography and subsequent plasma etching of the 6 nm PtSi film grown on a Si substrate. (b) Schematic view of a junction (not to scale) showing the layout of the constriction in the Hall bridge. (c) Scanning electron micrograph of the sample with 20 constrictions. (d) SEM subimage of the sample represented in (c). (e) The layout of a chain of SNS junctions showing the dimensions of the structure. Regions of the normal metal constrictions are dark, and the superconducting islands are light gray.
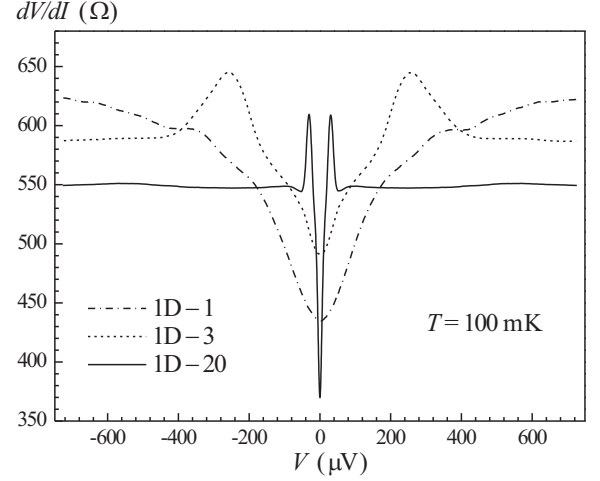
FIG. 2. Differential resistance vs dc bias voltage for samples: (1D-1) with single constriction, (1D-3) with three constrictions in series, and (1D-20) with twenty constrictions in series. $T = 100$ mK. Differential resistance and dc bias voltage are divided by 3 and 20 for samples 1D-3 and 1D-20, respectively.
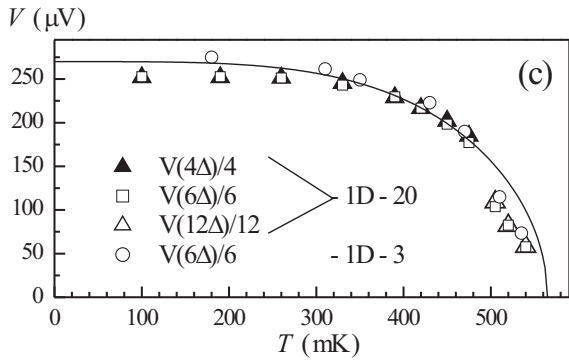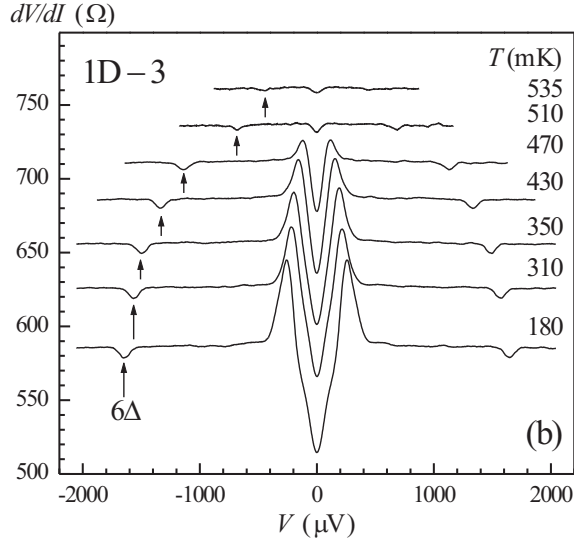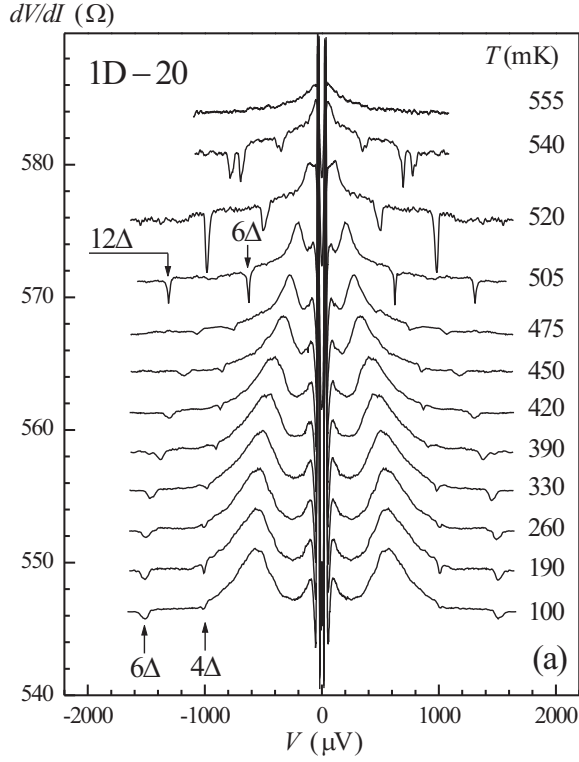
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FIG. 3. (a) Temperature evolution of the differential resistance of the sample 1D-20 as a function of the average bias voltage falling at one SNS junction. All traces except the lowest trace are shifted up for clarity. (b) The same for the sample 1D-3. The differential resistance reveals symmetrical minima at voltages exceeding 2Δ. The arrows indicate the above-energy-gap structure (AGS), corresponding to integer multiples of the 2Δ. The temperature dependence of the AGS at $eV = m \cdot 2\Delta$ divided by m for both samples is depicted in (c) by symbols, and compared to the BCS temperature dependence of the gap (solid line).
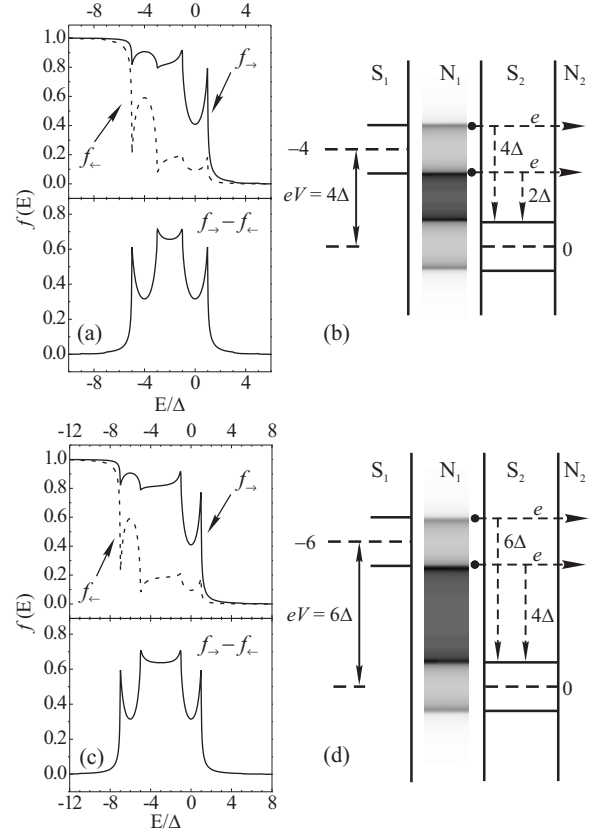

FIG. 4. Nonequilibrium distributions in normal region, for SNS junction with $Z = 0$: (a,b) at $eV = 4\Delta$; (c,d) at $eV = 6\Delta$ and $T = 0$. In (b) and (d) the function $[f_{\rightarrow}(E) - f_{\leftarrow}(E)]$ is depicted by the blackness (high values are black). The energy scale is plotted vertically.